Question 2. (10 points) Le random balls Suppose that you have 12 identical balls and 3 empty boxes. You randomly put every ball into some box. What is the probability that none of the boxes have more than 6 balls?

Raw work showing bars and stars is not the correct way to solve this problem:

———————— Summary of my Argument ——————————————

1. The color of the box (identical or not), would change the number of combinations possible but it would not change the likelihood of a ball being put in a box. The same logic applies to the balls, it doesn’t matter if the balls are identical or not. The probability of the ball landing in a box will not change because that ball is blue, only the number of combinations would change.

2. The law of large numbers should give an answer very close to the correct answer.

3. (Total number of combinations that meet condition / total combinations) does not equal (all possible ways an event can happen that meet the condition / all possible ways).

4. This is because not all combinations have the same probability of happening.

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Logical Reasoning for Solution (why bars and stars is not correct):

The problem with bars and stars is there are actually 4 placements possible.

Stars and bars cannot be used in that way because P(11) != P(02)

P(11) = 0.5 (two placements)

P(02) = 0.25 (one placement)

P(20) = 0.25 (one placement)

Using stars and bars means there are only three outcomes possible. But that doesn’t mean those outcomes have the same probability of happening.

P(11) has two ways it can happen.

Box1 Box2

B 1. 0

B. 0. 1

B. 0 1

B. 1. 0

P(20) has one way of happening

B 0 1

B. 0 1

P(20) has one way of happening

B 1. 0

B. 1. 0

Therefore, P(20) = P(02) = 1/4 chance of happening and P(11) has a 0.5 chance of happening.

If you made a possibility tree it would be

^

01. 10

^ ^

01 10 01 10

(02) (11) (11) (20)

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Explanation:

Here is another way to think about how the stars and bars combinations have different probabilities of happening. Let's say you have 2 boxes (box R (red), box B(blue)) and you have two identical balls. When you drop the first ball in you would expect it to have a 1/2 probability of landing in each box. You would also expect the second ball to have a 1/2 probability of landing in each box.

RB (First ball: Red, Second Ball: Blue)

RR (First ball: Red, Second Ball: Red)

BR (First ball: Blue, Second Ball: Red)

BB (First ball: Blue, Second Ball: Blue)

However, with the stars and bars logic it would be:

~~RB (First ball: Red, Second Ball: Blue)~~

RR (First ball: Red, Second Ball: Red)

BR (First ball: Blue, Second Ball: Red)

BB (First ball: Blue, Second Ball: Blue)

If the first ball goes in the red box, then the second ball goes in the red box.

Yet, this doesn’t make any sense. Why would the first ball change the outcome of the second ball?

Or maybe:

RB (First ball: Red, Second Ball: Blue) 1/6

RR (First ball: Red, Second Ball: Red) 1/3

BR (First ball: Blue, Second Ball: Red) 1/6

BB (First ball: Blue, Second Ball: Blue) 1/3

If the first ball goes in the red box, then there is a 2/3 chance the second ball goes in the red box.

Again, this doesn’t make any sense. Why would the first ball change the outcome of the second ball?

I suppose the theory is the balls are identical so what you would really see is:

RR (Ball: Red, Ball: Red) 1/3

RB (Ball: Blue, Ball: Red) 1/3

BB (Ball: Blue, Ball: Blue) 1/3

However, it doesn’t quite work this way, what actually happens is:

RR (Ball: Red, Ball: Red) 1/4

RB (Ball: Blue, Ball: Red) 1/2

BB (Ball: Blue, Ball: Blue) 1/4

What error was made with the identical balls?

“The errors in the incorrect computation for the probability are easily explained. The [balls], were assumed to be identical. This means that we cannot distinguish between any particular outcome, and the alternative outcome with their states exchanged. The mistake, then, is to consider these two indistinguishable outcomes as the same event. This leads to under-counting the space of possible outcomes and, together with the assumption that all outcomes have the same probability, gives the incorrect results. - <https://almostsuremath.com/2020/11/16/quantum-coin-tossing/>

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Example of how color changes number of combinations but not the probability of the event occurring

**Probability Number of balls in boxes are less than 3, Using Combinations (Wrong)**

Number of combinations for: P(300) = 1, P(030) = 1, P(003) = 1

P(300) = 1/27 != 1/10

BR BG BB (Red green blue boxes)

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

P(003) = 1/27 != 1/10

B -> 0 0 1

B -> 0 0 1

B -> 0 0 1

P(030) = 1/27 != 1/10

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

Answer for (Total number of combinations that meet condition / total combinations) = 7/10 // WRONG

Number of combination for 3 balls in one box, 0 in the rest, with identical boxes = 1

P(3 balls in one box) = 3/27 != 1/3

Total number of combinations 3.

Bx Bx Bx

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

P(003) = 1/27

B -> 0 0 1

B -> 0 0 1

B -> 0 0 1

P(030) = 1/27

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

Answer for (Total number of combinations that meet condition / total combinations) = 2/3 // WRONG

The odds of a ball landing in a box should not change because of the color of the box.

The answer would not change because (all possible outcomes that meet condition / all possible outcomes) does not change depending on the color of the box, the correct answer that the boxes have less than 3 is always = 0.88888888

————————————— Example with 3 Balls ——————————————

Each outcome has a (1/27 chance of occurring):

Possible Outcomes:

P(300) = 1/27

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

P(003) = 1/27

B -> 0 0 1

B -> 0 0 1

B -> 0 0 1

P(030) = 1/27

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

P(120) = 3/27

B -> 1 0 0

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

B -> 1 0 0

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

B -> 1 0 0

P(102) = 3/27

B -> 1 0 0

B -> 0 0 1

B -> 0 0 1

B -> 0 0 1

B -> 1 0 0

B -> 0 0 1

B -> 0 0 1

B -> 0 0 1

B -> 1 0 0

P(201) = 3/27

B -> 0 0 1

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

B -> 0 0 1

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

B -> 0 0 1

P(021) = 3/27

B -> 0 0 1

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

B -> 0 0 1

B -> 0 1 0

B -> 0 1 0

B -> 0 1 0

B -> 0 0 1

P(201) = 3/27

B -> 0 0 1

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

B -> 0 0 1

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

B -> 0 0 1

P(210) = 3/27

B -> 0 1 0

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

B -> 0 1 0

B -> 1 0 0

B -> 1 0 0

B -> 1 0 0

B -> 0 1 0

P(111) = 6/27

B -> 1 0 0

B -> 0 1 0

B -> 0 0 1

B -> 1 0 0

B -> 0 0 1

B -> 0 1 0

B -> 0 1 0

B -> 1 0 0

B -> 0 0 1

B -> 0 0 1

B -> 1 0 0

B -> 0 1 0

B -> 0 1 0

B -> 0 0 1

B -> 1 0 0

B -> 0 0 1

B -> 0 1 0

B -> 1 0 0

———— Bars and Stars code (WRONG) —————

#Incorrect code that others used to solve this problem. Not my code.

def distrib(n, k): #not my code

if k == 1:

return [[n]]

else:

rec=[]

for i in range(n+1):

lrec = distrib(i, k-1)

llrec=[[n-i]+j for j in lrec]

rec += llrec

return rec

print(distrib(3,3))

[[3, 0, 0], [2, 1, 0], [2, 0, 1], [1, 2, 0], [1, 1, 1], [1, 0, 2], [0, 3, 0], [0, 2, 1], [0, 1, 2], [0, 0, 3]]

WRONG BECASUE P([3, 0, 0]) != P([1, 1, 1])

——————— Example 2 balls ————————

I can show this in a very simple way with two balls and two boxes.

You randomly put the two balls in two boxes, box a (red) and box b (yellow). What are the odds a box has less than two balls?

**Simple logic / Common Sense:** First ball goes in, then there is 1 ball in one box and 0 in the other box. The odds of the ball going in the box with one ball is a 50 50 chance. P = 0.5.

Number of combinations

11

20

02

Correct answer: 1-P(02)+P(20) = 0.5

Stars and bars: 1/3 = 0.33333 (Stars and bars cannot be used in this way)

Law of large numbers with 1,000,000 trials gets: 0.498993

—————————————————————————— Ways it can happen —————————————————————————

P(11) = 0.5

a b (Red and yellow)

1 0

0 1

0 1

1 0

P(02) = 0.25

0 1

0 1

P(20) = 0.25

1 0

1 0